行政院國家科學委員會專題研究計畫　期中進度報告

線性及非線性貝龍--佛羅貝紐斯理論(1/3)

計畫類別：個別型計畫
計畫編號：NSC92-2115-M-032-006-
執行期間：92 年 08 月 01 日至 93 年 07 月 31 日
執行單位：淡江大學數學系

計畫主持人：譚必信

計畫參與人員：國科會助理　謝旻宜；　洪劍能

報告類型：精簡報告
報告附件：出席國際會議研究心得報告及發表論文
處理方式：本計畫可公開查詢

中　華　民　國 93 年 5 月 17 日

行政院國家科學委員會補助專題研究計畫期中進度報告

線性及非線性貝龍–佛羅貝紐斯理論 (1/3)
Linear and nonlinear Perron-Frobenius theory

計畫類別: ☑ 個別型計畫　☐ 整合型計畫
計畫編號: NSC 92–2115–M–032–006
執行期間: 92 年 8 月 1 日　至　93 年 7 月 31 日

計畫主持人: 譚必信
共同主持人:
計畫參與人員: 國科會助理　謝旻宜
　　　　　　　國科會助理　洪劍能

成果報告類型 (依經費核定清單規定繳交): 期中精簡報告

本成果報告包括以下應繳交之附件:
☐ 赴國外出差或研習心得報告一份
☐ 赴大陸地區出差或研習心得報告一份
☐ 出席國際學術會議心得報告及發表之論文各一份
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執行單位: 淡江大學數學系

中　華　民　國　93 年 5 月 17 日

Intermediate Report on
Linear and nonlinear Perron-Frobenius theory

In the past year, I have been working mainly on the following:

Problem 4. Continue some of my not completed work on the spectral theory of linear cone-preserving maps and study the open problems given in my review paper [T5].

The above is the fourth problem of the four problems proposed in my three-year research project. In particular, I have completed (and submitted for publication) a long paper entitled "The Perron generalized eigenspace and the spectral cone of a cone-preserving map". This investigation has stretched over a period of more than ten years. Formerly, it beard the title "On semipositive bases for a cone-preserving map" and was given as [T6] in the bibliography of this three-year research project. As considerably more new material has been found, the title is altered to reflect the change in contents.

Let me quote from the abstract of my paper:

"A unified treatment is offered to reprove known results on the following four highlights of the combinatorial spectral theory of nonnegative matrices, or to extend (or partly extend) the results to the setting of a linear map preserving a polyhedral proper (or proper) cone: the preferred-basis theorem, equivalent conditions for equality of the (graph-theoretic) level characteristic and the (spectral) height characteristic, the strong majorization relation between the two characteristics, and the relation between the combinatorial properties of a nonnegative matrix and the positivity of the individual entries in its principal components. This is achieved by employing the new concept of spectral cone of a cone-preserving map and combining the cone-theoretic methods developed in our previous papers on the geometric spectral theory of cone-preserving maps with the algebraic-analytic method introduced by Hartwig, Neumann and Rose and further exploited by Neumann and Schneider for nonnegative matrices."

The paper consists of the following nine sections:

1. Introduction
2. Preliminaries
3. Nonnegative components and K-semipositive bases
4. The spectral cone
5. Equality of the height and the level characteristics
6. A cone-theoretic proof of the preferred-basis theorem
7. The strong majorization relation between the level and the height characteristics
8. Principal components of a nonnegative matrix
9. Final remarks

Below is a sample of results obtained in my paper:

Theorem 1. *Let K be a proper cone in $\mathbb{R}^n$, and let $A \in \pi(K)$. For any nonnegative integer $k = 0, \ldots, \nu_{\rho(A)}(A) - 1$, consider the following conditions*:

(a) $Z_A^{(k)} \in \pi(K)$.

(b) $J_A^{(k)}(\varepsilon) \in \pi(K)$ *for all positive* ε (*or, for all sufficiently large* ε).

(c) $J_A^{(k)}(\varepsilon) \in \pi(K)$ *for all sufficiently small positive* ε.

(d) $J_A^{(k)}(\varepsilon) \in \pi(K)$ *for at least one positive* ε.

(e) $\mathcal{R}((\rho(A)I - A)^k) \cap E(A)$ *contains a K-semipositive basis.*

Then conditions (a) *and* (b) *are equivalent, conditions* (c) *and* (d) *are also equivalent, and the following implications hold:* (a) $\Longrightarrow$ (c) $\Longrightarrow$ (e).

In the above, $\pi(K)$ stands for the set of all matrices A such that $AK \subseteq K$. We use $Z_A^{(k)}$ to denote the kth principal component of A, i.e., $Z_A^{(k)} = (A - \rho(A)I)^k Z_A^{(0)}$, where $Z_A^{(0)}$ is the projection of $\mathbb{C}^n$ onto $E(A)$ $(:= \mathcal{N}((\rho(A)I - A)^n)$, the Perron generalized eigenspace of A, along the direct sum of other generalized eigenspaces of A. We also use $J_A^{(k)}(\varepsilon)$ to denote the kth transform principal component of A, i.e.,

$$J_A^{(k)}(\varepsilon) = Z_A^{(k)} + Z_A^{(k+1)}/\varepsilon + \cdots + Z^{(\nu_\rho - 1)}/\varepsilon^{\nu_\rho - k - 1},$$

where ν_ρ stands for the index of $\rho(A)$ as an eigenvalue of A.

Theorem 2. *Let K be a polyhedral proper cone in $\mathbb{R}^n$, and let $A \in \pi(K)$. Then*:

(i) *For* $k = 0, \ldots, \nu_{\rho(A)}(A) - 1$, $J_A^{(k)}(\varepsilon) \in \pi(K)$ *for all sufficiently small positive* ε.

(ii) $\varepsilon^{-1} J_A^{(0)}(\varepsilon) + R(0) \in \pi(K)$ *for all sufficiently small positive* ε, *where* $R(\varepsilon)$ *denotes the analytic operator that appears in* (2.1).

Theorem 3. *Let K be a polyhedral proper cone and let $A \in \pi(K)$. Then:*

(i) *$C(A,K)$ is polyhedral.*

(ii) *$J_A^{(0)}(\varepsilon)(E(A)\cap K)$ (and also $J_A^{(0)}(\varepsilon)K$) is a full subcone of $C(A,K)$ for all sufficiently small $\varepsilon > 0$.*

(iii) *For $i = 1,\ldots,\nu_\rho$, $\mathcal{N}((A-\rho(A)I)^i)\cap C(A,K)$ is an $(A-\rho(A)I)$-invariant face of $C(A,K)$ and also a full subcone of $\mathcal{N}((A-\rho(A)I)^i)\cap K$. In particular, $C(A,K)$ is a full subcone of $E(A)\cap K$ and hence a proper cone in $E(A)$.*

(iv) *For $i = 0,1,\ldots,\nu_\rho-1$ and for sufficiently small $\varepsilon > 0$, $J_A^{(i)}(\varepsilon)K$ (also $J_A^{(i)}(\varepsilon)(E(A)\cap K)$) is an A-invariant subcone of $C(A,K)$ and also a full cone in $\mathcal{R}((A-\rho(A)I)^i)\cap E(A)$.*

In the above, $C(A,K)$ stands for the spectral cone of A, i.e., the set $\{x \in K\colon (A-\rho(A)I)^i x \in K\ \forall i \in \mathbb{Z}_+\}$.

For any $A \in \pi(K)$, the height characteristic $\eta(A) = (\eta_1,\ldots,\eta_p)$ and level characteristic $\lambda(A) = (\lambda_1,\ldots,\lambda_{\nu_p})$ of A are defined respectively by:

$$\begin{aligned} \eta_k &= \dim\mathcal{N}((\rho(A)I-A)^k) - \dim\mathcal{N}((\rho(A)I-A)^{k-1}), \\ \text{and}\quad \lambda_k &= \dim\operatorname{span}[\mathcal{N}((A-\rho(A)I)^k)\cap K] - \dim\operatorname{span}[\mathcal{N}((A-\rho(A)I)^{k-1})\cap K]. \end{aligned}$$

Theorem 4. *Let K be a proper cone, and let $A \in \pi(K)$. Consider the following conditions:*

(a) *$\eta(A) = \lambda(A)$.*

(b) *$\eta(A) = \xi(A)$.*

(c) *Every vector in $E(A)$ is a peak vector.*

(d) *For each k, $k = 1,\ldots,\nu_\rho$, $\mathcal{N}((A-\rho(A)I)^k)$ contains a K-semipositive basis.*

(e) *There exists a K-semipositive height basis for A.*

(f) *There exists a K-semipositive height-level basis for A.*

(g) *There exists a K-semipositive Jordan basis for A.*

(h) *For each k, $k = 1,\ldots,\nu_\rho$, $\mathcal{N}((A-\rho(A)I)^k)\cap C(A,K)$ is a full cone in $\mathcal{N}((A-\rho(A)I)^k)$.*

(i) *For each k, $k = 1,\ldots,\nu_\rho$, we have*

$$\eta_k(A) = \dim(A-\rho(A)I)^{k-1}[\mathcal{N}((A-\rho(A)I)^k)\cap C(A,K)].$$

Conditions (a)–(f) *are equivalent and so are conditions* (g)–(i)*. Moreover, we always have* (g) $\Longrightarrow$ (a)*, and when K is polyhedral, conditions* (a)–(i) *are all equivalent.*

Our proof of the Hershkowitz-Richman-Rothblum-Schneider preferred-basis theorem (for a nonnegative matrix) relies on the following cone-theoretic result:

Theorem 5. *Let K be a polyhedral proper cone, and let $A \in \pi(K)$. If K is a semi-distinguished A-invariant face of itself, then $\Phi((A-\rho(A)I)C(A,K)) = \Phi(\mathcal{N}((A-\rho(A)I)^{\nu-1}) \cap K)$, where $\nu = \nu_{\rho(A)}(A)$.*

For finite sequences $\alpha = (\alpha_1, \ldots, \alpha_p)$, $\beta = (\beta_1, \ldots, \beta_p)$, we write $\alpha \prec \beta$ if $\sum_{i=1}^{k} \alpha_i \leq \sum_{i=1}^{k} \beta_i$ for $k = 1, \ldots, p-1$, and $\sum_{i=1}^{p} \alpha_i = \sum_{i=1}^{p} \beta_i$. The following is an extension of the known strong majorization relation between the level characteristic and the height characteristic of a nonnegative matrix.

Theorem 6. *Let K be a polyhedral proper cone and let $A \in \pi(K)$. Then $\widehat{\lambda(A)} \prec \eta(A)$, where $\widehat{\lambda(A)}$ denotes the finite sequence obtained from $\lambda(A)$ by reordering its terms in a nonincreasing order.*

If $A \in \pi(K)$ and F is a face of K, then we denote by $\widehat{F}$ the smallest A-invariant face of K including F.

Theorem 7. *Let K be a polyhedral proper cone and let $A \in \pi(K)$. Let $\varepsilon > 0$ be such that $J_A^{(0)}(\varepsilon) \in \pi(K)$. Then for any $0 \neq x \in K$, we have*

(i) $\Phi(\widehat{J_A^{(0)}(\varepsilon)x}) = \Phi(E(A) \cap \widehat{\Phi(x)})$; *and*

(ii) $\operatorname{span} \Phi(\widehat{J_A^{(0)}(\varepsilon)x}) = W_{Z_A^{(0)}x}$, *where W_x denotes the A-invariant subspace generated by x.*

附表二十一

專題研究計畫補助費延期及變更申請對照表

<table>
<tr><td>執　行
機　構</td><td colspan="2">淡江大學數學系</td><td>計　畫
編　號</td><td colspan="2">NSC 93-2115-M-032-001</td></tr>
<tr><td>計　畫
名　稱</td><td colspan="5">線性及非線性貝龍—佛羅貝紐斯理論(2/3)</td></tr>
<tr><td>合　約
期　限</td><td colspan="2">自 93 年 8 月 1 日起至 94 年 7 月 31 日止</td><td>希望延
長期限</td><td colspan="2">至 94 年 7 月 31 日止如期結案</td></tr>
<tr><td colspan="3">原　核　定　補　助　情　形</td><td colspan="3">擬申請變更用途及金額情形</td></tr>
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<tr><td>延期或變更用途說明</td><td colspan="5">原指導的博士生已畢業，另將指導兩位升博二(尚未通過資格考試)的研究生，姓名分別為吳淑惠及張定中。(按現行核給標準，博二生給七單位獎助金)

人事費：博士班研究生獎助金增加為 168 獎助單位(原為 60 單位)。
故申請增加 216,000 元(108 獎助單位)，合計為 948,888 元</td></tr>
</table>

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傳　　真：(02)26209916

（本表請填送一式二份，如不敷填寫可仿此格式放大印用）